Applications to Flux Compactifications

Ex: gravity on a group manifold

$(G, \cdot)$ multiplication group
differentiable manifold

$$g \in G \longmapsto \Lambda_L(h)\, g \equiv h g ,$$

$$g \in G \longmapsto \Lambda_R(h)\, g \equiv g h .$$

basis of left-invariant 1-forms $\{\sigma^m\}$

$$T_m \sigma^m = g^{-1} dg ,$$

where $\{T_m\}$ are the generators of $\mathfrak{g}$ and $d \equiv dy^m \frac{\partial}{\partial y^m}$. On a coordinate basis,

$$\sigma^m = U^m{}_n(y)\, dy^n .$$

$$ds^2_{10} = ds^2_D + \underbrace{g_{mn}\, \sigma^m \otimes \sigma^n}_{\substack{(10-D)\text{-dimensional} \\ \text{group } G}}$$

Theorem (Maurer-Cartan)

$$\boxed{\begin{array}{l} \omega_{mn}{}^p \equiv -2 (U^{-1})^q{}_m (U^{-1})^r{}_n\, \partial_{[q} U^p{}_{r]} \\[1em] \text{are constants and represent the structure constants} \\ \text{of } \mathfrak{g}.\ (\text{realised on } G\text{'s Killing vectors}) \end{array}}$$

The resulting gravity theory in D dimensions has G as a gauge symmetry and the resulting deformation parameter $\omega_{mn}{}^{p}$ is usually called METRIC FLUX .

Ex : Gauge Fluxes in type IIA supergravity

NS - NS 2 - form : B_2

R - R odd p - forms : $C_1, C_3, C_5, C_7, (C_{-1}, C_9)$

Modified field strengths : $H_3 = dB_2$,

$$G_{p+1} = dC_p - dB_2 \wedge C_{p-2} .$$

When reducing $\underbrace{x^{\hat{\mu}}}_{10} \longmapsto (\underbrace{x^{\mu}}_{D}, \underbrace{y^{m}}_{10-D})$,

one can integrate out the y - dependence inside the full Lagrangian.

If the internal manifold $\mathcal{M}_{10-D}$ has non-trivial (p + 1) - cycles, $(\mathcal{C}_{p+1})$

$$\int_{\mathcal{C}_{p+1}} G_{p+1} \neq 0 \quad \equiv F_{p+1}$$

→ I say that I have a gauge flux

GAUGE FLUXES also appear as deformation parameters in my reduced theory.

→ One can couple them to local sources (Dp - branes, Op - planes, etc.) through tadpoles.

Ex : D6 - branes in IIA compactifications

$$10 \longrightarrow 4$$

$$\overbrace{\underbrace{\times \ \times\times\times}_{4D} \ \vdots \ \times\times\times}^{\text{worldvolume } X_7} \underbrace{\ \ - - -}_{y^m}$$

$$S_{DBI} = -T \int_{X_7} \underbrace{\sqrt{-\tilde{g}_7}}_{\substack{\text{pull-back} \\ \text{of } g_{10} \text{ on } X_7}} d^7\xi \, e^{-\phi} = -T \int_{X_7} e^{-\phi} \, vol_7$$

$$S_{WZ} = \int_{X_7} Q_{D6} C_7 = \int_{10} C_7 \wedge \underbrace{(\text{tadpole})_3}_{\substack{\text{cancelled by} \\ \text{quadratic expressions} \\ \text{in the fluxes}}}$$

Tadpole cancellation

$$\left\{ \begin{matrix} \text{Quadratic in} \\ \text{the fluxes} \end{matrix} \right\} = \left\{ \begin{matrix} \text{Sources} \\ D6/O6 \end{matrix} \right\} \quad \text{(modified Bianchi identities)}$$

At the end of the day,

$$ds_{10}^2 = \tau^{-2} ds_4^2 + \rho \underbrace{M_{mn}}_{SL(6)/SO(6)} dy^m \otimes dy^n$$

(ρ, τ) combination of vol_6 and 10D ϕ.

$$\int d^{10}x \sqrt{-g_{10}} \, R^{(10)} e^{-2\phi} \longrightarrow \int d^4x \sqrt{-g_4} \left(R^{(4)} + \underbrace{\rho^{-1} R^{(6)}}_{\substack{\text{scalar potential} \\ (\text{fct of } M_{mn}, \\ \text{quadratic in } \omega_{mn}{}^p)}} \right)$$

$$\left(R^{(6)} \propto - \omega_{mn}{}^p \omega_{qr}{}^s M^{mq} M^{nr} M_{ps} + \right.$$
$$\left. + \cdots \right)$$

$$\boxed{M^{mn} \equiv (M^{-1})^{mn}}$$

$$\int d^{10}x \sqrt{-g_{10}}\, e^{-2\phi} |H_3|^2 \longrightarrow$$

$$\longrightarrow \int d^4x \sqrt{-g_4}\, \underbrace{\rho^{-3} \tau^{-2} H_{mnp} H_{qrs} M^{mq} M^{nr} M^{ps}}_{\text{another piece of a scalar potential !}}$$

$V(\rho, \tau, M_{mn})$: quadratic in fluxes

Generalised BI $\rightarrow$ quadratic constraints for fluxes

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$\mathcal{N} = 4$ gauging & type II orientifolds

D = 4 half-maximal supergravity

$$G_0 = SL(2) \times SO(6,6)$$

$V = (2, 12)$ (e-m duality relates half of them to the others)

$$\mathrm{adj}(G_0) = (3,1) \oplus (1,66)$$

$$\xrightarrow{LC} \quad \Theta \in \underbrace{(2,12)}_{\xi_{\alpha M}} \oplus \underbrace{(2,220)}_{f_{\alpha [MNP]}}$$

$\xi_{\alpha M}$: SL(2) gaugings

$f_{\alpha[MNP]}$: purely $\subset SO(6,6)$ gaugings

$$\begin{cases} \alpha : \square \; SL(2) \\ M : \square \; SO(6,6) \end{cases}$$

Let's restrict to $\xi_{\alpha M} = 0$ (more subtle).

Θ satisfies the following QC

$$\begin{cases} f_{\alpha R[MN} f_{\beta PQ]}{}^{R} = 0 \,, & \text{(Jacobi in the } (3, \text{▯})\text{)} \\ \varepsilon^{\alpha\beta} f_{\alpha MNR} f_{\beta PQ}{}^{R} = 0 \,. & \text{(antisymmetry of [,] in the } (1, \text{⊞})\text{)} \end{cases}$$

$(\xi_{\alpha M} = 0)$

Scalars

$$M_{\alpha\beta} \in \frac{SL(2)}{SO(2)} \quad (2 \text{ scalars}) \,,$$

$$M_{MN} \in \frac{SO(6,6)}{SO(6) \times SO(6)} \quad (36 \text{ scalars}) \,.$$

$\rightarrow$ 38 in total.

$$V = \frac{1}{64} f_{\alpha MNP} f_{\beta QRS} M^{\alpha\beta} \left(\frac{1}{3} M^{MQ} M^{NR} M^{PS} + \left(\frac{2}{3} \eta^{MQ} - M^{MQ} \right) \eta^{NR} \eta^{PS} \right)$$
$$- \frac{1}{144} f_{\alpha MNP} f_{\beta QRS} \varepsilon^{\alpha\beta} M^{MNPQRS} \qquad (\xi_{\alpha M} = 0) \,,$$

where $M_{MN} = \mathcal{V}_M{}^{\underline{m}} \mathcal{V}_N{}^{\underline{n}} \quad (\underline{m} : SO(6)_{\text{timelike}})$,

$$M_{MNPQRS} \equiv \varepsilon_{\underline{m}_1 \cdots \underline{m}_6} \mathcal{V}_M{}^{\underline{m}_1} \cdots \mathcal{V}_S{}^{\underline{m}_6} \,,$$

η_{MN} LC $SO(6,6)$ metric.

Gauge Algebra (G)

$$[T_{\alpha M}, T_{\beta N}] = f_{\alpha MN}{}^{P} T_{\beta P}$$

Note that $f_{\alpha MNP}$ are generalised structure constants in the sense that $\{T_{\alpha M}\}$ are 24 generators, but only 12 of them are independent!

Type IIA w/ O6/D6 sources

$$\text{O6}: \quad \times \; \times\times\times \,\vdots\, \underbrace{\times\times\times}_{\substack{a \\ \text{(even)}}} \; \underbrace{- - -}_{\substack{i \\ \text{(odd)}}}$$

This defines an orientifold $\mathbb{Z}_2$ involution that <u>breaks</u> SUSY from 32 to 16 ($\mathcal{N} = 4$) in $D = 4$.
Also $SL(6) \subset SO(6,6)$ gets broken into $\mathbb{R}^+ \times SL(3)_a \times SL(3)_i$.

$$\Theta \in (2, 220) \text{ of } SL(2)_S \times SO(6,6)$$

$$\longrightarrow \quad \text{-----} \quad \text{of} \quad \mathbb{R}^+_S \times \mathbb{R}^+_T \times SL(6)$$

$$\longrightarrow \quad \text{-----} \quad \text{of} \quad \mathbb{R}^+_S \times \mathbb{R}^+_T \times \mathbb{R}^+_U \times SL(3)_a \times SL(3)_i$$

inside here I will recognise my fluxes by matching the corresponding <u>representations</u>

<u>e.g.</u> $\omega_{mn}{}^{p} \in \underbrace{\boxplus}_{84}$ of $SL(6)$

$$\rightarrow \underbrace{\omega_{ai}{}^{j}}_{(3',8)} \oplus \underbrace{\omega_{ab}{}^{c}}_{(6',1)} \oplus \underbrace{\cancel{\omega_{ab}{}^{k} \oplus \omega_{ij}{}^{k}}}_{\mathbb{Z}_2\text{ - odd !}}$$

$$\left.\begin{array}{l} F_0 \\ F_2 \\ F_4 \\ F_6 \end{array}\right\} \text{R - R gauge fluxes} \qquad \left.\begin{array}{l} H_3 \\ \omega \end{array}\right\} \text{NS - NS fluxes}$$

→ One finds the following <u>dictionary</u> :

Θ components	IIA fluxes	Ex. Sol. (isotropic)
$f_{+\bar{a}\bar{b}\bar{c}}$	F_{aibjck}	$\frac{3\sqrt{10}}{2}$
$f_{+\bar{a}\bar{b}\bar{k}}$	F_{aibj}	$\sqrt{6}/2$
$f_{+\bar{a}\bar{j}\bar{k}}$	F_{ai}	$-\frac{\sqrt{10}}{6}$
$f_{+\bar{i}\bar{j}\bar{k}}$	F_0	$\frac{5\sqrt{6}}{6}$
$f_{-\bar{a}\bar{b}\bar{c}}$	H_{ijk}	$-\frac{\sqrt{6}}{3}$
$f_{-\bar{a}\bar{b}\bar{k}}$	$\omega_{ij}{}^{c}$	$\frac{\sqrt{10}}{3}$
$f_{+\bar{a}\bar{b}k}$	H_{abk}	$\frac{\sqrt{6}}{3}$
$f_{+\bar{a}\bar{j}k}$	$\omega_{ak}{}^{j}$	$\sqrt{10}$
$f_{+a\bar{b}\bar{c}}$	$\omega_{bc}{}^{a}$	$\sqrt{10}$

↓ non-trivial QC :

$$\omega^2 = 0 \quad \text{(No KK5 !)}$$

$$(F_0 H_3 + F_2 \cdot \omega)^{\perp} = 0 \quad \text{(No D6/O6 in } \perp \text{ dir. !)}$$

<u>Not</u> seen as a constraint by $\mathcal{N}=4$ supergravity (SUSY - allowed sources)

$$(F_0 H_3 + F_2 \cdot \omega)^{\parallel} = N_6 \quad (\text{combination of } N_{D6} \,\&\, N_{O6})$$

<u>Ex</u> : In the above solution, $V_0 = -1$ (AdS)

$N_6^{\parallel} = N_6^{\perp} = 0$, SUSY & STABLE

(IIA on $SU(2) \times SU(2)$ w/ R-R & NS-NS gauge fluxes and no local sources) → $\boxed{\text{non-semisimple gauging in } \mathcal{N}=4 \text{ sugra}}$